COMP3153/9153
Homework 4

Bounded Model Checking, Timed Automata, TCTL

Due: April 27, 2020, 11:59am (Sydney Time)
Submission guidelines are given at the end of this document.

Exercise 1 (Bounded Model Checking) (20 Marks)

The automaton $\mathcal{A}$ of Fig. 1 depicted below has 5 states, and 0 is the initial state.

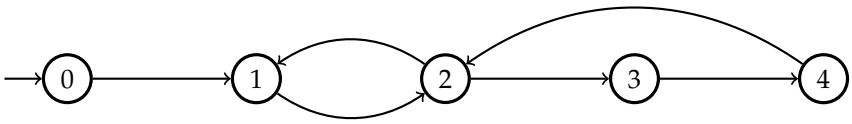

Figure 1: Automaton $\mathcal{A}$

Assume we encode the states in binary, using a vector $x[n]$ of size n (components range from 0 to $n-1$).

Question 1 How many digits, n, do we need to encode the 5 states?

We want to encode the transition relation of $\mathcal{A}$ (Fig. 1) using the encoding introduced in the lecture slides. We let $T(x, x')$ be the encoded transition relation.

Question 2 Give the propositional formula that defines $T(x, x')$.
Hint: In case there is more than one outgoing transition from a state, use the *or* ($\vee$) connector.

As presented in the lectures, the *diameter* of an automaton A is the least upper bound (or maximum) of the shortest distances between any *connected* states.

Question 3 What is the diameter of $\mathcal{A}$?

Let $at(j)$ denote the atomic proposition that states that automaton $\mathcal{A}$ is in state j. Let $\mathsf{For}(\varphi)$ be defined as given for finite paths in the lecture slides.

Question 4 Give the value of $\mathsf{For}(\mathtt{at}(4))_{\mathtt{i}}, 0 \leqslant \mathtt{i} \leqslant 4$ for $\mathtt{k} = 4$.

Exercise 2 (Timed Automata) (30 Marks)

A *snack vending machine* is specified as follows:

1. Initially, it is ready to accept a coin;
2. After a coin is inserted, three choices can be made by the customer: *coffee*, *ice cream*, or *cancel* (which means the customer gets their money back).
3. After a choice of coffee or ice cream is made, the vending happens between 8 and 10 time units;
4. If the choice is *cancel*, the money is given back no earlier than 2 time units, and no later than 5 time units after the choice happened;
5. If no choice is made within 15 time units after coin insertion, the order is cancelled and the money is given back to the customer. The customer does not need to do anything: the machine returns the money after 15 time units if no order has been issued by the customer;
6. After the order is delivered or cancelled, the machine returns to the initial state (ready to accept a coin.)

Assume that the events/actions for the vending machine are from the set:

$\{\mathtt{coin\_in}, \mathtt{coin\_back}, \mathtt{coffee}, \mathtt{ice\_cream}, \mathtt{cancel}, \mathtt{vend\_coffee}, \mathtt{vend\_ice\_cream}\}$

Question 5 Give a timed automaton, $\mathcal{D}$, that represents the behaviours of the vending machine as specified above.
Note: Explain how many clocks you need and why you need them as well as the purpose of any invariants you use.

Question 6
a) Is there a run in $\mathcal{D}$, where coin_in and coin_back are more than 16 time units apart?
b) What happens if the customer issues an order, *coffee* or *ice cream*, exactly 15 time units after inserting a coin?

Exercise 3 (Region Automata) (30 Marks)

Assume a timed automaton C given in Figure 2.

Question 7 On a 2-dimensional grid, draw the regions needed to build the region graph/automaton for C.

Question 8 Construct the region graph/automaton of C.

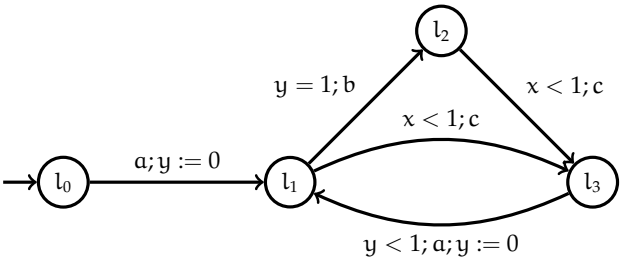

Figure 2: Timed automaton C

Question 9 Use the constructed automaton to decide whether l_3 is reachable; give an execution leading to l_3. (It is not necessary to give precise times, use variables $t_1, t_2, \ldots$ for the durations in the execution.)

Exercise 4 (TCTL and TCTL$_C$) (20 Marks)

Note: p, q, r, `crash` and `airbag` are atomic propositions.

Assume the following TCTL formula Ψ

$$\Psi := \mathbf{EG}_{\geqslant 0}\big((\mathbf{AG}_{\leqslant 10} p \vee \mathbf{EF}_{=10} \neg p) \Rightarrow \mathbf{AF}_{\leqslant 2} r\big)$$

Question 10 Describe in plain English the meaning of Ψ.

Question 11 Give a TCTL$_C$ formula that is equivalent to Ψ.

Question 12 Give a (Büchi) timed automaton that accepts the language defined by the timed words that satisfy the formula:

$$\mathbf{AG}_{\geqslant 15}(\texttt{crash} \Rightarrow \mathbf{AF}_{<1}\texttt{airbag}) \,.$$

Submission Guidelines

- Due time: April 27, 2020, 11:59am (Sydney Time). **No late submission allowed**.
- Submit one PDF file (`hw4.pdf`) using the CSE `give` system by typing the command `give cs3153 hw4 hw4.pdf` on a CSE terminal. Alternatively use the online submission page.
- It is highly recommended that you use LaTeX to prepare your document. A guide is provided on the course website.
